

Two-Field Quintom Models in the $w - w'$ Plane

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The $w - w'$ plane, defined by the equation of state parameter for the dark energy and its derivative with respect to the logarithm of the scale factor, is useful to the study of classifying the dynamical dark energy models. In this note, we examine the evolving behavior of the two-field quintom models with w crossing the $w = -1$ barrier in the $w - w'$ plane. We find that these models can be divided into two categories, type A quintom in which w changes from > -1 to < -1 and type B quintom in which w changes from < -1 to > -1 as the universe expands.

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Recent observations of type Ia supernovae suggest that the expansion of the universe is accelerating and that two-thirds of the total energy density exists in a dark energy component with negative pressure [1]. In addition, measurements of the cosmic microwave background [2] and the galaxy power spectrum [3] also indicate the existence of the dark energy. The simplest candidate for the dark energy is a cosmological constant Λ , which has pressure $P_\Lambda = -\rho_\Lambda$. Specifically, a reliable model should explain why the present amount of the dark energy is so small compared with the fundamental scale (fine-tuning problem) and why it is comparable with the critical density today (coincidence problem). The cosmological constant suffers from both these problems. One possible approach to constructing a viable model for dark energy is to associate it with a slowly evolving and spatially homogeneous scalar field ϕ , called “quintessence” [4, 5, 6]. Such a model for a broad class of potentials can give the energy density converging to its present value for a wide set of initial conditions in the past and possess tracker behavior (see, e.g., [7] for a review).

Quintessence models describe the dark energy with a time-varying equation of state parameter, w , the ratio of its pressure to energy density and $w > -1$. Recently, such models have been extended to phantom dark energy with $w < -1$ [8] (see for example [9] and references therein). The physical sources for phantom fields with strongly negative pressure may be looked for in string theory [10] and supergravity [11]. Such fields may arise from quantum effects on a locally de Sitter background [12]. They may also be present in modified gravity theories, such as higher order theories [13] and scalar-tensor theories [14]. Coupled quintessence with dark matter may also lead to $w < -1$ [15]. Phantom field with a negative kinetic term may be a simplest implementing, in which the weak energy condition is violated. It has been shown that such models possess the attractor

behavior similar to quintessence models [16].

If $w < -1$ in an expanding universe, the energy density of the dark energy increases with time, which leads to unwanted future singularity called “big rip” [17]. Thus from this point of view the transition from $w > -1$ to $w < -1$ or vice versa would be desirable for the history of the universe. On the other hand, the analysis on the properties of dark energy from the recent observations mildly favors models with w crossing -1 in the near past. From the theoretical viewpoint, it is necessary to explore possibilities for dark energy with w crossing -1 . However, neither quintessence nor phantom can fulfill this transition. The similar conclusion has also been obtained for the k-essence models [18]. Quintom models easily provide a way to realize this transition [19, 20]. The quintom fields may be associated with some higher derivatives terms [21] derived from fundamental theories, for instance due to the quantum corrections or the non-local physics in the string theory [22]. They may also arise from a slowly decaying D3-brane in a local effective approximation [23]. Interestingly, the quintom models differ from the quintessence or phantom in evolution and the determination of the fate of universe [24]. There exist lots of interests in the literature presently in building of quintom-like models [25], such as hessence models [26] and brane models [27].

Recently, Caldwell and Linder examined the evolving behavior of quintessence models of dark energy in the $w - w'$ phase plane, where w' is the time derivative of w with respect to the logarithm of the scale factor a , and showed that these models occupy the thawing and freezing regions in the phase plane [28]. More recently, these results were extended to a more general class of quintessence models with a monotonic potential [29] and phantom dark energy [30]. In this note we extend these studies with single-field quintessence or phantom models to two-field quintom models. Our results show that there exist two types of quintom models according to the evolving behavior around $w = -1$. Moreover, we plot the trajectories numerically for the two types in the $w - w'$ plane.

Let us consider the following model which contains a

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negative-kinetic scalar field ϕ (phantom) and a normal scalar field ψ (quintessence):

$$S = \int d^4x \sqrt{-g} \left[\frac{R}{2\kappa^2} - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \frac{1}{2} g^{\mu\nu} \partial_\mu \psi \partial_\nu \psi + V(\phi, \psi) + \mathcal{L}_m \right], \quad (1)$$

where $\kappa^2 \equiv 8\pi G_N$ the gravitational coupling, $V(\phi, \psi)$ the scalar potential and \mathcal{L}_m the Lagrangian density of matter fields. In a flat FRW cosmology the evolutions of the fields are governed by

$$\ddot{\phi} + 3H\dot{\phi} - V_{,\phi} = 0, \quad (2)$$

$$\ddot{\psi} + 3H\dot{\psi} + V_{,\psi} = 0, \quad (3)$$

where $V_{,\phi} = \partial V / \partial \phi$ and $V_{,\psi} = \partial V / \partial \psi$. In what follows we assume that there is no direct coupling between the phantom field and the normal scalar field, i.e., $V(\phi, \psi) = V_\phi(\phi) + V_\psi(\psi)$. Then the effective equation of state w is given by

$$w = \frac{-\dot{\phi}^2 + \dot{\psi}^2 - 2V}{-\dot{\phi}^2 + \dot{\psi}^2 + 2V}, \quad (4)$$

$$= \frac{\Omega_\phi w_\phi + \Omega_\psi w_\psi}{\Omega_{DE}}, \quad (5)$$

where $\Omega_{DE} = \Omega_\phi + \Omega_\psi$ is the density parameter of dark energy, $w_\phi = (\dot{\phi}^2 + 2V_\phi)/(\dot{\phi}^2 - 2V_\phi)$ and $w_\psi = (\dot{\psi}^2 - 2V_\psi)/(\dot{\psi}^2 + 2V_\psi)$. For a model with a normal scalar field, the equation of state $w \geq -1$. The toy model of a phantom energy component with a negative kinetic term possesses an equation of state $w < -1$. In our model, Eq. (5) implies $w \geq -1$ when the quintessence component ψ dominates the density of the universe and $w < -1$ when the phantom component ϕ dominates. By using the equations of motion (2) and (3), the derivative of (4) with respect to $\ln a$ can be rewritten as [5]

$$w' = 3(1-w)^2 \frac{\dot{\phi}^2 - \dot{\psi}^2}{2V} - (1-w) \frac{1}{V} (V_{,\phi} \frac{\dot{\phi}}{H} + V_{,\psi} \frac{\dot{\psi}}{H}). \quad (6)$$

By using the following relations

$$\begin{aligned} \frac{\dot{\phi}^2 - \dot{\psi}^2}{2V} &= -\frac{1+w}{1-w}, \quad \frac{\kappa \dot{\phi}}{H} = \pm \sqrt{-3(1+w_\phi)\Omega_\phi}, \\ \frac{\kappa \dot{\psi}}{H} &= \pm \sqrt{3(1+w_\psi)\Omega_\psi}, \end{aligned} \quad (7)$$

the expression for w' becomes

$$w' = -3(1-w)^2 + (1-w) \frac{1}{\kappa V} \left[\pm V_{,\phi} \sqrt{-3(1+w_\phi)\Omega_\phi} \pm V_{,\psi} \sqrt{3(1+w_\psi)\Omega_\psi} \right]. \quad (8)$$

For a phantom field ϕ climbing up its potential, the \pm sign before $V_{,\phi}$ depends on whether $V_{,\phi} < 0$ or $V_{,\phi} > 0$, respectively. For a quintessence field ψ rolling down its potential, the \pm sign before $V_{,\psi}$ corresponds to $V_{,\psi} > 0$ or $V_{,\psi} < 0$, respectively.

According to the evolving behavior of w around -1 , the two-field quintom models of dynamical dark energy are classified into the following two types: type A quintom characterized by w from $w > -1$ to $w < -1$ and type B quintom characterized by w from $w < -1$ to $w > -1$.

Type A quintom models:

In such models, the equation of state changes from $w > -1$ to $w < -1$, i.e., the universe evolves from a quintessence-dominated phase to a phantom-dominated phase. Therefore, the properties of the late-time attractor solution are determined by the phantom potential $V_\phi(\phi)$. The cosmological evolution of the Type A quintom model with

$$V(\phi, \psi) = V_{\phi 0} e^{-\lambda_\phi \kappa \phi} + V_{\psi 0} e^{-\lambda_\psi \kappa \psi} \quad (9)$$

was investigated in detail in Ref. [20]. When the phantom component is dominated at late times, Eq. (8) reduces to

$$w' = (1-w) \left[-3(1+w) - \lambda_\phi \sqrt{-3(1+w)} \right], \quad (10)$$

which has three critical points $w = 1$, $w = -1$ and $w = -1 - \lambda_\phi^2/3$. It is easily shown that the scaling solution $w = -1 - \lambda_\phi^2/3$ is the stable attractor of this type of models, i.e., the ratio of kinetic to potential energy of the phantom field becomes a constant. The top panels of Fig. 1 shows the evolving behavior of the models in the $w - w'$ phase plane. We shall next consider a general case in which the effective equation of state w tends to -1 at late times. In this case, the ratio of kinetic to potential energy tends to zero [16]. The features of the behavior are virtually independent of the precise shape of the quintessence potential since the contribution of the quintessence component becomes negligible at late times compared to the phantom component. For example we consider a positive power-law potential, which have been previously investigated in Refs. [16, 31]. For the following potential

$$V(\phi, \psi) = V_{\phi 0} \phi^\alpha + V_{\psi 0} \psi^\alpha, \quad (11)$$

in the $\phi \rightarrow \infty$ limit, we have $w' = -3(1-w)(1+w)$. The critical point with $w = -1$ is the late-time attractor, i.e., the quintom field becomes ultimately frozen, as shown in the bottom panels of Fig. 1.

Type B quintom models:

In such models, the equation of state changes from $w < -1$ to $w > -1$. In principle, for the two-field system consisting of quintessence and phantom, if the phantom initially dominates the universe, it will still dominate up to future, and thus the universe can not exit the phantom phase ($w < -1$) forever. We take a simple potential

$$V(\phi, \psi) = V_{\phi 0} e^{-\lambda_\phi \phi^2} + V_{\psi 0} e^{-\lambda_\psi \psi^2} \quad (12)$$

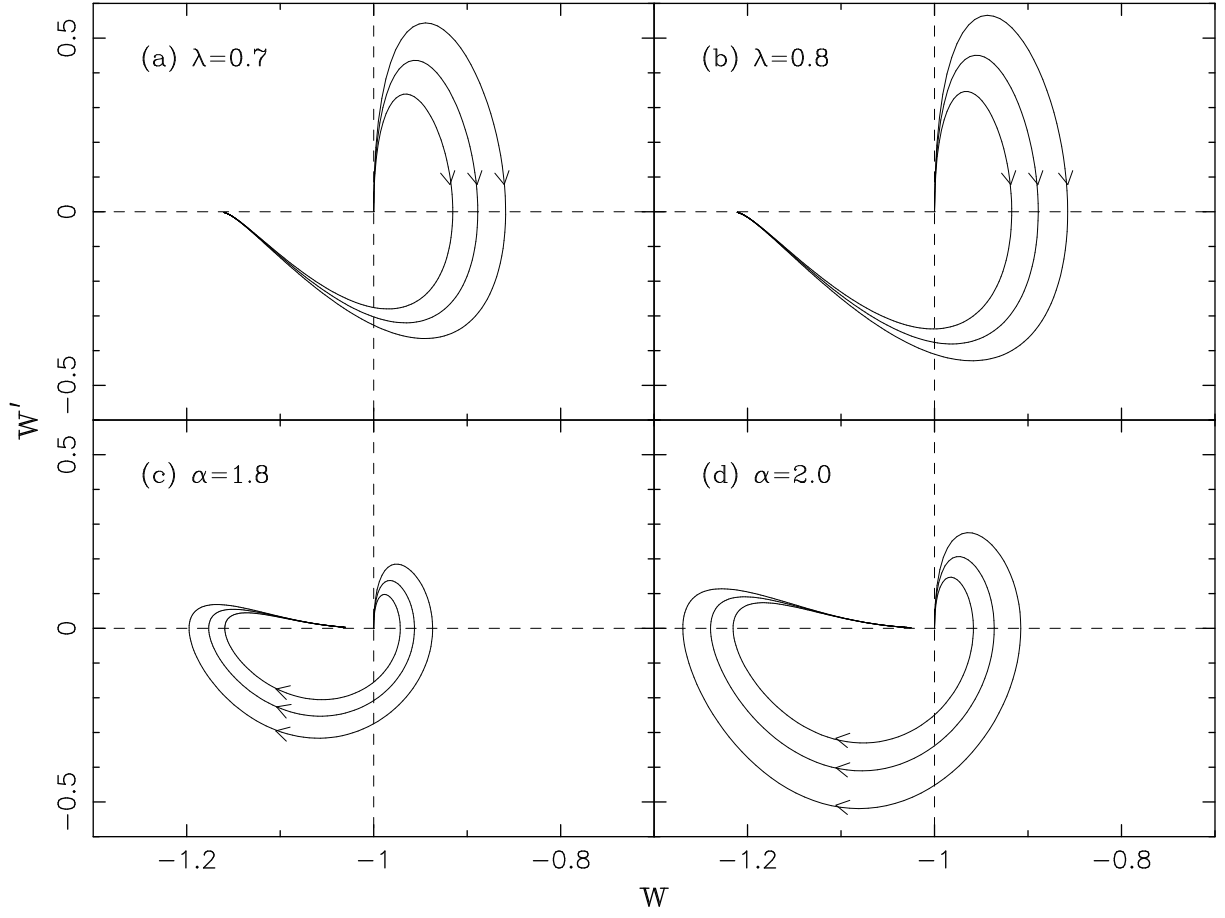


FIG. 1: The evolution of the type A quintom models of dynamical dark energy in the $w - w'$ phase plane. The tracks in panels (a), (b), (c) and (d) correspond to the quintom models with the potentials $V(\phi, \psi) = V_{\phi 0} e^{-\lambda_{\phi} \phi} + V_{\psi 0} e^{-\lambda_{\psi} \psi}$ ($\lambda_{\phi} = 0.7, 0.8$) and $V(\phi, \psi) = V_{\phi 0} \phi^{\alpha} + V_{\psi 0} \psi^{\alpha}$ ($\alpha = 1.8, 2.0$), respectively.

for a illustration. The relevant figure is plotted in the bottom panels of Fig. 2. In general the phantom field will climb up its potential, which makes its energy density increasing constantly during its evolution. Thus in order to implement the change of equation of state from $w < -1$ to $w > -1$, a naive choice is taking the potential of phantom field zero, for example, considering the following potential

$$V(\phi, \psi) = V_{\psi 0} e^{-\lambda_{\psi} \psi}, \quad (13)$$

whose evolving behavior is plotted in the top panels of Fig. 2. In this case, the initial kinetic energy of phantom field is not zero, which naturally leads to the state equation $w < -1$ of quintom system. But with the expansion of the universe, the kinetic energy of phantom field will become negligible gradually while the energy of quintessence field will begin to dominate the universe. Thus we have $w > -1$. However, it should be noted that here the energy of phantom field is negative, but the total energy of quintom system is still positive. We do not want to deeply discuss the relevant physics with this case, and we here only emphasize that to obtain such

a transition from $w < -1$ to $w > -1$ in two-field system (quintessence+phantom), the above condition seems to be necessarily satisfied.

In conclusion, it is well-known that the two-field quintom models give a simplest realization to the dynamical dark energy with the equation of state parameter crossing the $w = -1$ barrier, thus it is very interesting to explore the full evolving behavior of various two-field quintom models. In this note, we have examined the evolving behavior of the two-field quintom models in the $w - w'$ plane. We find that these models can generally be divided into two categories, type A quintom models in which w changes from > -1 to < -1 and type B quintom models in which w changes from < -1 to > -1 , which has not been analyzed in detail before. Compared to the latter, the former is easily constructed since the energy density of the phantom field increases as the universe expands and ultimately dominates the universe. The latter requires the phantom field with a flat potential or a potential with a maximum.

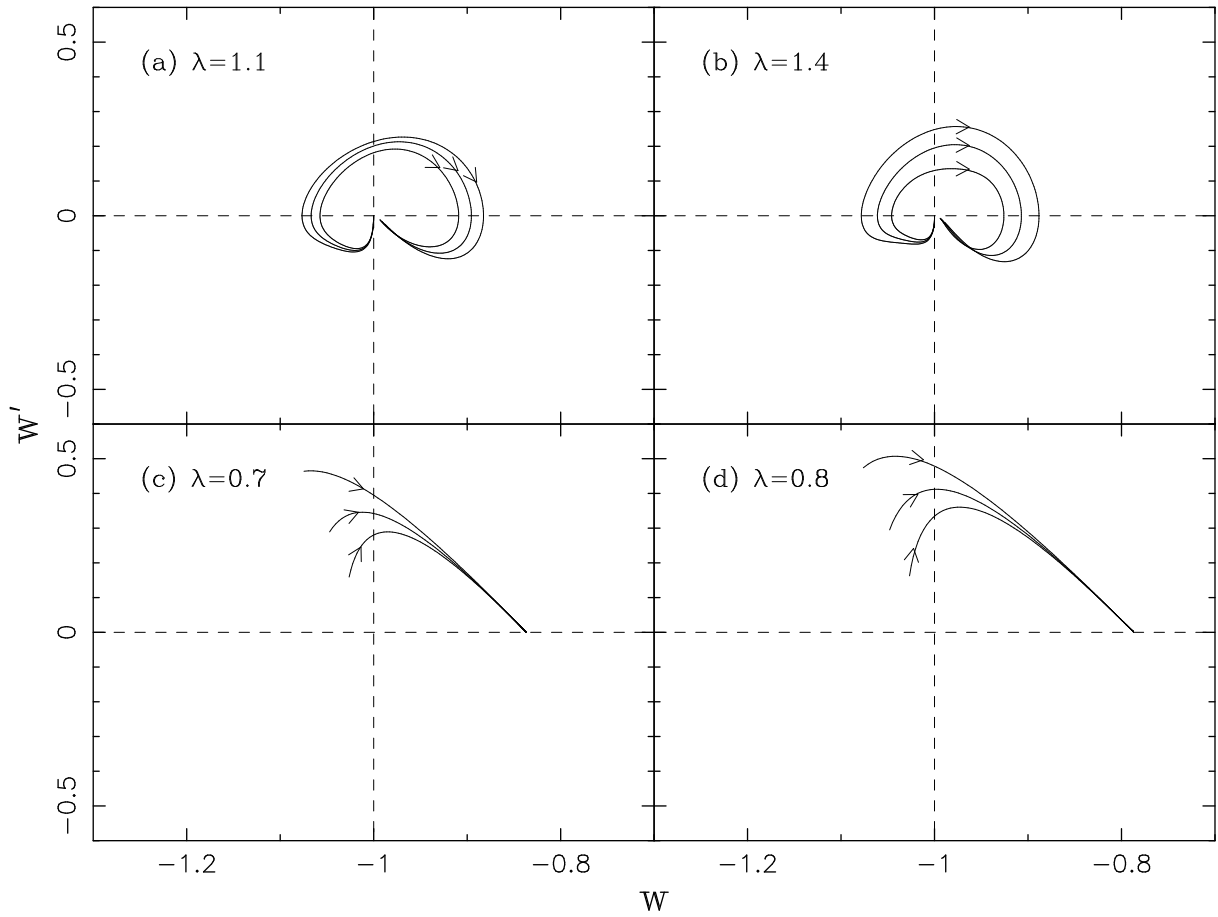


FIG. 2: The evolution of the type B quintom models of dynamical dark energy in the (w, w') phase plane. The tracks in panels (a), (b), (c) and (d) correspond to the quintom models with the potentials $V(\phi, \psi) = V_{\phi 0} e^{-\lambda_{\phi} \phi^2} + V_{\psi 0} e^{-\lambda_{\psi} \psi^2}$ ($\lambda_{\psi} = 1.1, 1.4$) and $V(\phi, \psi) = V_0 e^{-\lambda \psi}$ ($\lambda = 0.7, 0.8$), respectively.

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- [1] A.G. Riess et al., *Astron. J.* **116** (1998) 1009; S. Perlmutter et al., *Astrophys. J.* **517** (1999) 565.
 - [2] D.N. Spergel et al., *Astrophys. J. Suppl.* **148** (2003) 175.
 - [3] M. Tegmark et al., *Phys. Rev.* **D69** (2004) 103501.
 - [4] B. Ratra and P.J.E. Peebles, *Phys. Rev.* **D37** (1988) 3406; C. Wetterich, *Nucl. Phys.* **B302** (1988) 668; R. Caldwell, R. Dave and P.J. Steinhardt, *Phys. Rev. Lett.* **80** (1998) 1582.
 - [5] I. Zlatev, L. Wang and P.J. Steinhardt, *Phys. Rev. Lett.* **82** (1999) 896; P.J. Steinhardt, L. Wang and I. Zlatev, *Phys. Rev.* **D59** (1999) 123504.
 - [6] P.G. Ferreira and M. Joyce, *Phys. Rev.* **D58** (1998) 023503; P. Binetruy, *Phys. Rev.* **D60** (1999) 063502; P. Brax and J. Martin, *Phys. Lett.* **B468** (1999) 40; P. Binetruy, *Int. J. Theor. Phys.* **39** (2000) 1859; P. Brax, J. Martin and A. Riazuelo, *Phys. Rev.* **D62** (2000) 103505; S.M. Carroll, *Phys. Rev. Lett.* **81** (1998) 3067; C. Kolda and D.H. Lyth, *Phys. Lett.* **B458** (1999) 197.
 - [7] V. Sahni and A. Starobinsky, *Int. J. Mod. Phys.* **D9** (2000) 373; J.E. Peebles and B. Ratra, *Rev. Mod. Phys.* **75** (2003) 559; T. Padmanabhan, *Phys. Rept.* **380** (2003) 235; E.J. Copeland, M. Sami and S. Tsujikawa, *hep-th/0603057*.
 - [8] R.R. Caldwell, *Phys. Lett.* **B545** (2002) 23.
 - [9] S. Nojiri and S.D. Odintsov, *Phys. Lett.* **B562** (2003) 147; S. Nojiri and S.D. Odintsov, *Phys. Lett.* **B565** (2003) 1; P. Singh, M. Sami and N. Dadhich, *Phys. Rev. D* **68** (2003) 023522; R.G. Cai and A. Wang, *JCAP* **0503** (2005) 002; Y.S. Piao and E. Zhou, *Phys. Rev. D* **68** (2003) 083515; Y.S. Piao and Y.Z. Zhang, *Phys.*

- Rev. **D70** (2004) 063513; F.S.N. Lobo, Phys. Rev. **D71** (2005) 084011; F.S.N. Lobo, Phys. Rev. **D71** (2005) 124022; H.M. Sadjadi, Phys. Rev. **D73** (2006) 063525; J.M. Aguirregabiria, L.P. Chimento and R. Lazkoz, Phys. Rev. **D70** (2004) 023509; L.P. Chimento and R. Lazkoz, Int. J. Mod. Phys. **D14** (2005) 587; L.A. Urena-Lopez, JCAP **0509** (2005) 013; L. Amendola, S. Tsujikawa and M. Sami, Phys. Lett. **B632** (2006) 155; A.A. Coley, S. Hervik and W.C. Lim, gr-qc/0512085; A.A. Coley, S. Hervik, J. Latta, astro-ph/0503169; M. Kaplinghat and A. Rajaraman, astro-ph/0601517; F.S.N. Lobo, gr-qc/0603091; C.J. Gao and S.N. Zhang, hep-th/0604114; V.A. Rubakov, hep-th/0604153; Z.G. Huang, H.Q. Lu and W. Fang, hep-th/0604160; A.K. Sanyal, astro-ph/0605388.
- [10] P.H. Frampton, Phys. Lett. **B555** (2003) 139.
- [11] H.P. Nilles, Phys. Rep. **110** (1984) 1.
- [12] V.K. Onemli and R.P. Woodard, Class. Quant. Grav. **19** (2002) 4607; V.K. Onemli and R.P. Woodard, Phys. Rev. **D70** (2004) 107301.
- [13] M.D. Pollock, Phys. Lett. **B215** (1988) 635.
- [14] D.F. Torres, Phys. Rev. **D66** (2002) 043522; V. Faraoni, Phys. Rev. **D68** (2003) 063508; L. Perivolaropoulos, JCAP **0510** (2005) 001; S.M. Carroll et al., Phys. Rev. **D71** (2005) 063513; J. Martin, C. Schmid and J. Uzan, Phys. Rev. Lett. **96** (2006) 061303.
- [15] S. Das, P.S. Corasaniti and J. Khoury, Phys. Rev. **D73** (2006) 083509.
- [16] Z.K. Guo, Y.S. Piao and Y.Z. Zhang, Phys. Lett. **B594** (2004) 247.
- [17] R.R. Caldwell, M. Kamionkowski and N.N. Weinberg, Phys. Rev. Lett. **91** (2003) 071301; V. Faraoni and W. Israel, Phys. Rev. **D71** (2005) 064017; V. Faraoni, Class. Quant. Grav. **22** (2005) 3235; M. Bouhmadi-Lopez and J.A.J. Madrid, JCAP **0505** (2005) 005; S. Nojiri, S.D. Odintsov and S. Tsujikawa, Phys. Rev. **D71** (2005) 063004; S. Nojiri and S.D. Odintsov, Phys. Rev. **D72** (2005) 023003; M. Sami, A. Toporensky, P.V. Tretjakov and S. Tsujikawa, Phys. Lett. **B619** (2005) 193; P. Wu and H. Yu, Nucl. Phys. **B727** (2005) 355; S. Nesseris, L. Perivolaropoulos, Phys. Rev. **D70** (2004) 123529; M.P. Dabrowski and T. Stachowiak, Annals Phys. **321** (2006) 771; E.M. Barboza Jr. and N.A. Lemos, gr-qc/0606084; Y.H. Wei, astro-ph/0607359.
- [18] A. Vikman, Phys. Rev. **D71** (2005) 023515.
- [19] B. Feng, X.L. Wang and X. Zhang, Phys. Lett. **B607** (2005) 35.
- [20] Z.K. Guo, Y.S. Piao, X. Zhang and Y.Z. Zhang, Phys. Lett. **B608** (2005) 177.
- [21] M. Li, B. Feng and X. Zhang, JCAP **0512** (2005) 002.
- [22] J.Z. Simon, Phys. Rev. **D41** (1990) 3720; A. Elizer and R.P. Woodard, Nucl. Phys. **B325** (1989) 389; T.G. Erler and D.J. Gross, hep-th/0406199.
- [23] I.Ya. Aref'eva, A.S. Koshelev and S.Yu. Vernov, Phys. Rev. **D72** (2005) 064017.
- [24] B. Feng, M. Li, Y.S. Piao and X. Zhang, Phys. Lett. **B634** (2006) 101; J.Q. Xia, B. Feng and X. Zhang, Mod. Phys. Lett. **A20** (2005) 2409.
- [25] V. Sahni and Y. Shtanov, JCAP **0311** (2003) 014; H. Stefancic, Phys. Rev. **D71** (2005) 124036; J. Sola and H. Stefancic, Phys. Lett. **B624** (2005) 147; X.F. Zhang, H. Li, Y.S. Piao and X. Zhang, astro-ph/0501652. A.A. Andrianov, F. Cannata and A.Y. Kamenshchik, Phys. Rev. **D72** (2005) 043531; B. Wang, Y. Gong and E. Abdalla, Phys. Lett. **B624** (2005) 141; X. Zhang and F.Q. Wu, Phys. Rev. **D72** (2005) 043524; Z. Chang, F.Q. Wu and X. Zhang, Phys. Lett. **B633** (2006) 14; P. Wu and H. Yu, Int. J. Mod. Phys. **D14** (2005) 1873; H. Zhang and Z.H. Zhu, Phys. Rev. **D73** (2006) 043518; W. Zhao and Y. Zhang, Class. Quant. Grav. **23** (2006) 3405; W. Zhao and Y. Zhang, Phys. Rev. **D73** (2006) 123509; R. Lazkoz, G. Leon, astro-ph/0602590; F. Cannata and A.Y. Kamenshchik, gr-qc/0603129; L.P. Chimento and R. Lazkoz, astro-ph/0604090; M. Alimohammadi and H.M. Sadjadi, gr-qc/0608016.
- [26] H. Wei, R.G. Cai and D.F. Zeng, Class. Quant. Grav. **22** (2005) 3189; H. Wei and R.G. Cai, Phys. Rev. **D72** (2005) 123507; H. Wei and R.G. Cai, Phys. Lett. **B634** (2006) 9; H. Wei and R.G. Cai, Phys. Rev. **D73** (2006) 083002; M. Alimohammadi and H.M. Sadjadi, Phys. Rev. **D73** (2006) 083527.
- [27] G. Kofinas, G. Panotopoulos and T.N. Tomaras, JHEP **0601** (2006) 107; R.G. Cai, Y. Gong and B. Wang, JCAP **0603** (2006) 006; P.S. Apostolopoulos and N. Tetradis, Phys. Rev. **D74** (2006) 064021.
- [28] R.R. Caldwell and E.V. Linder, Phys. Rev. Lett. **95** (2005) 141301.
- [29] R.J. Scherrer, Phys. Rev. **D73** (2006) 043502.
- [30] T. Chiba, Phys. Rev. **D73** (2006) 063501.
- [31] M. Sami and A. Toporensky, Mod. Phys. Lett. **A19** (2004) 1509; L. Perivolaropoulos, Phys. Rev. **D71** (2005) 063503; H.M. Sadjadi and M. Alimohammadi, Phys. Rev. **D74** (2006) 043506; J. Kujat, R.J. Scherrer and A.A. Sen, Phys. Rev. **D74** (2006) 083501.